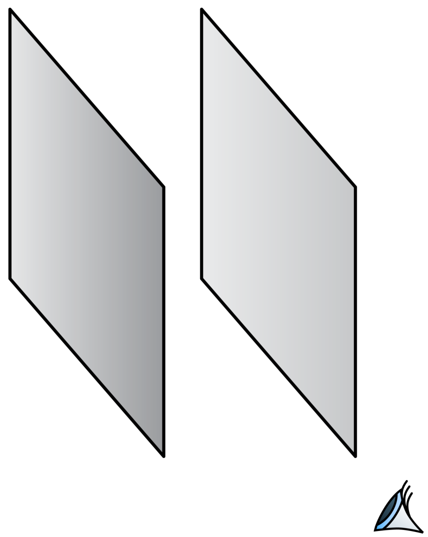
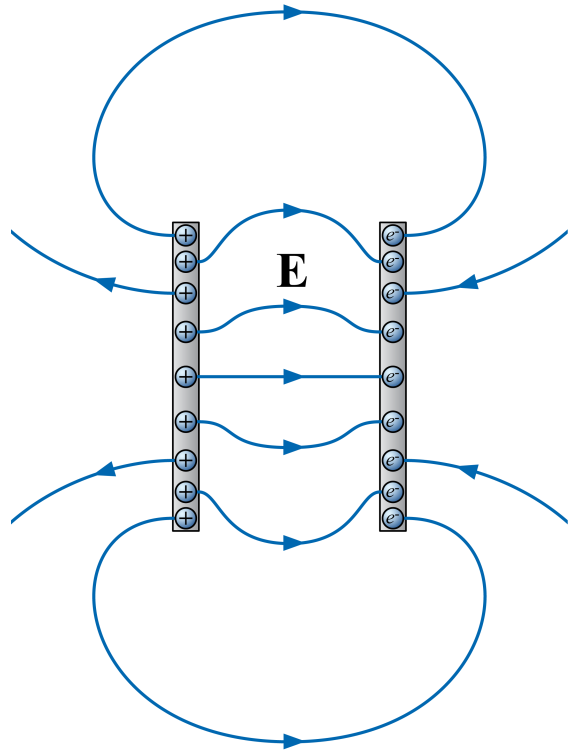
Capacitors

Simplifying sketches of electric field patterns for parallel plates

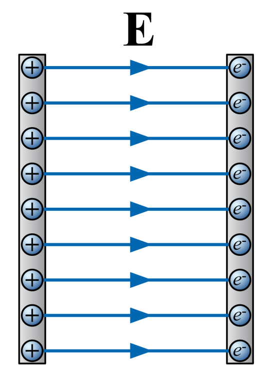
Conducting plates



Rough sketch of electric field pattern with some realistic features

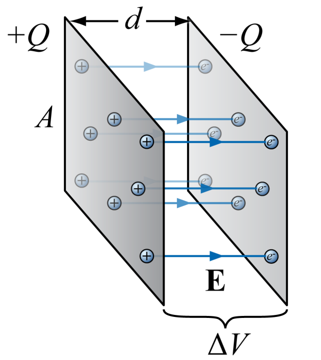


Simplified sketch of electric field pattern



Capacitors

capacitor – pair of conducting terminals across which a potential difference can be established by painting a positive charge on one of the terminals and a negative charge of equal magnitude on the other terminal

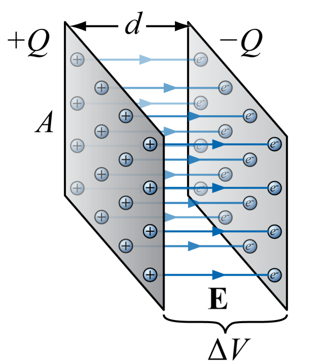


(Curved fringe field not shown)

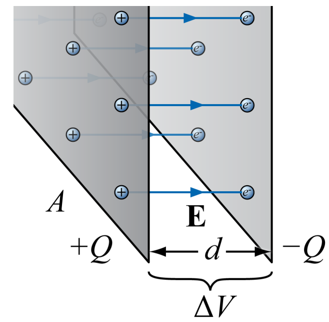
$$Q \Rightarrow \vec{\mathbf{E}}$$

$$\left|\vec{\mathbf{E}}_{\mathrm{AVG}}\right| = \frac{|\Delta V|}{\Delta \ell_{\parallel}}$$

Can the potential difference between the terminals be changed by changing the amount of charge painted on the terminals?



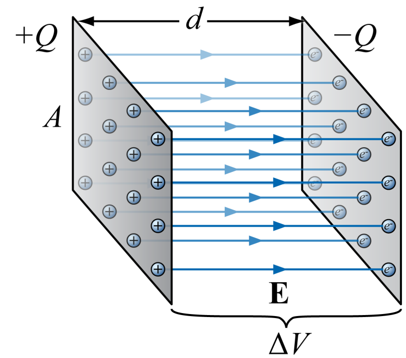
$$\uparrow Q \Rightarrow \uparrow \Delta V$$



$$\uparrow A \Rightarrow \downarrow \left|\vec{\mathbf{E}}\right| \Rightarrow \downarrow \Delta V$$

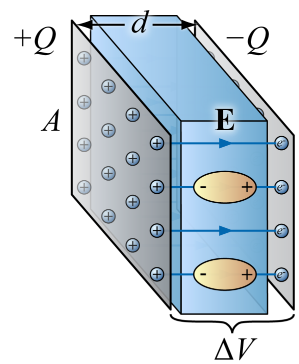
$$\uparrow A \Rightarrow \uparrow C$$

$$\left|\vec{\mathbf{E}}_{\parallel,\mathrm{VAC}}\right| = \frac{Q}{\epsilon_0 A}$$



$$\uparrow d \Rightarrow \uparrow \Delta V$$

$$\uparrow d \Rightarrow \downarrow C$$



$$\left|\vec{\mathbf{E}}\right| = \frac{\left|\vec{\mathbf{E}}_{\mathrm{VAC}}\right|}{\kappa}$$

$$\uparrow \kappa \Rightarrow \downarrow \left|\vec{\mathbf{E}}\right| \Rightarrow \downarrow \Delta V$$

$$\uparrow \kappa \Rightarrow \uparrow C$$

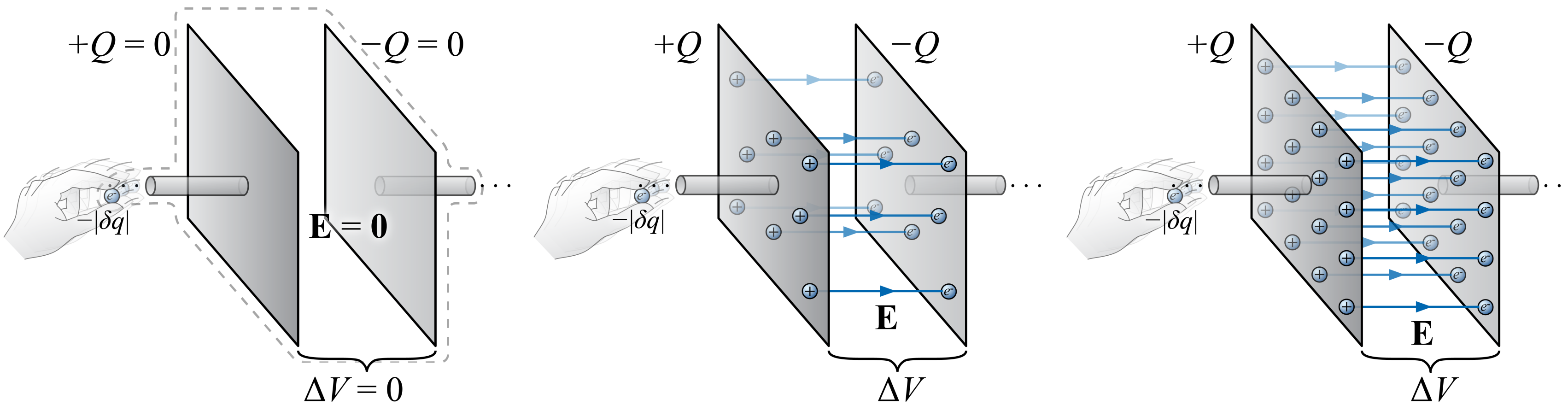
$$\Delta V = \frac{Q}{C}$$

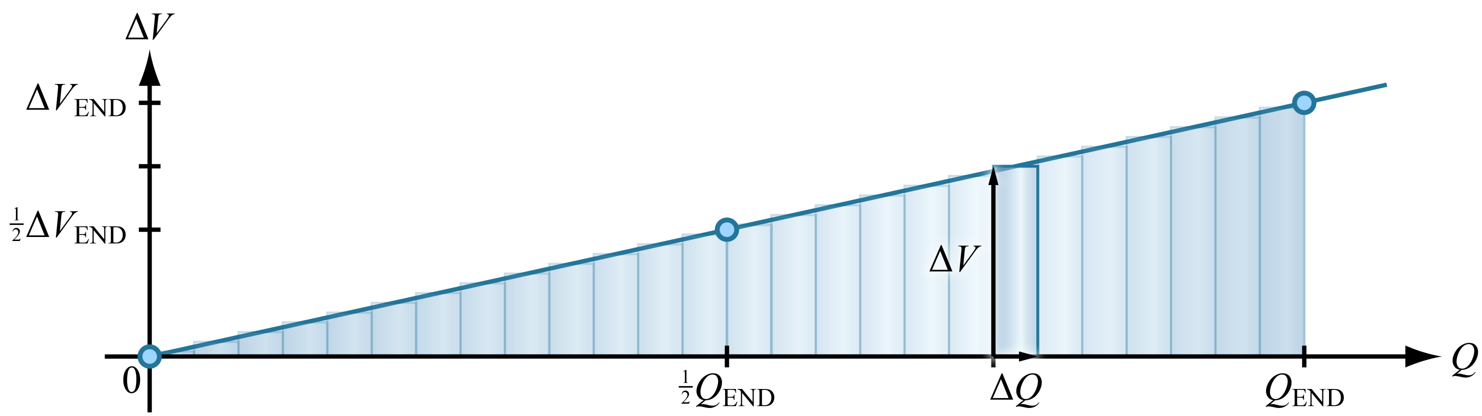
$$[C] = \frac{\mathrm{C}}{\mathrm{V}} = \mathrm{F}$$

$$C_{\parallel\ \mathrm{PLATES}} = \kappa \epsilon_0 \frac{A}{d}$$

Capacitors

Storing energy in a capacitor





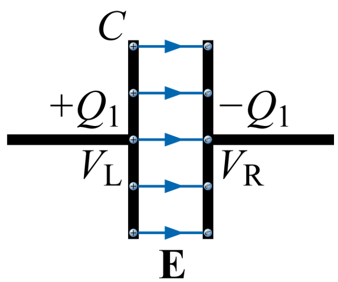
$$\Delta W_{\text{HAND}} = |\delta q|0 + \cdots + |\delta q|\left|\frac{1}{2}\Delta V_{\text{END}}\right| + \cdots + |\delta q||\Delta V_{\text{END}}| \underbrace{=}_{\text{APPC}} \int_{Q=0}^{Q=Q_{\text{END}}} \Delta V \, \mathrm{d}Q$$

$$\Delta U_{\text{C}} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}$$

Capacitors

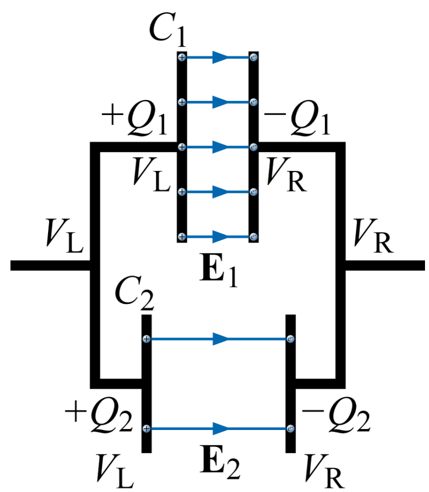
Capacitors in circuits

Single capacitor



$$\Delta V = \frac{Q}{C}$$

Capacitors in parallel

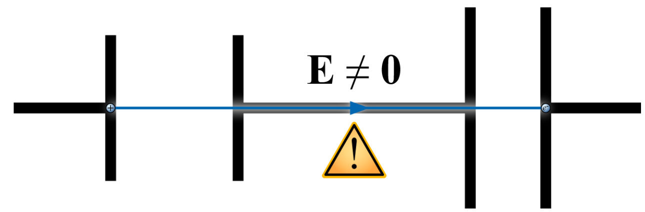


Same ΔV and $\uparrow Q \Rightarrow \uparrow C$

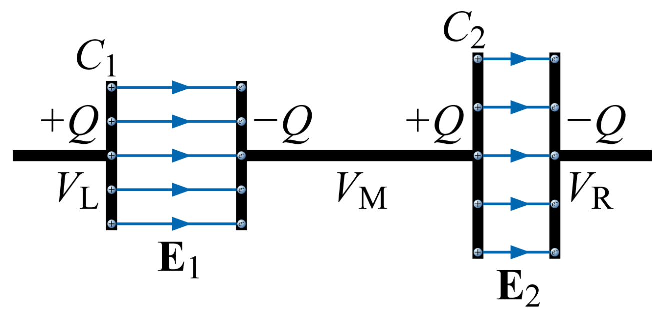
$$C_{\mathrm{EQ},\parallel} = C_1 + C_2 + \cdots$$

Q_{EQ} divides so that $Q_i \propto C_i$

Capacitors in series



$|\vec{\mathbf{E}}|$ should quickly approach 0 in conductor (quickly achieves electrostatic equilibrium)



Same Q and $\uparrow \Delta V \Rightarrow \downarrow C$

$$\frac{1}{C_{\mathrm{EQ,SER}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

ΔV_{EQ} divides so that $\Delta V_i \propto \frac{1}{C_i}$